Using FastMap to Solve Graph Problems in a Euclidean Space

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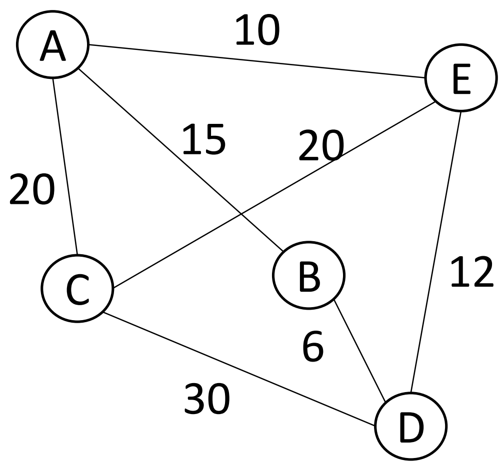
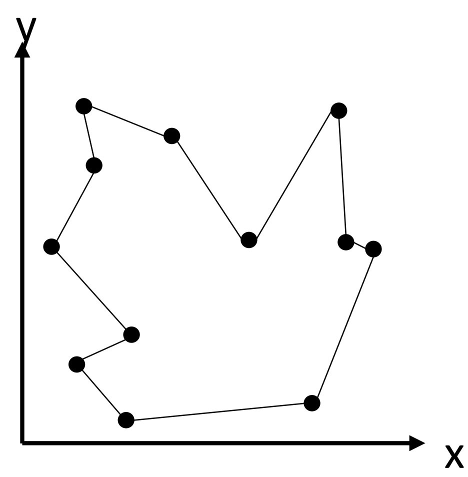
Motivation

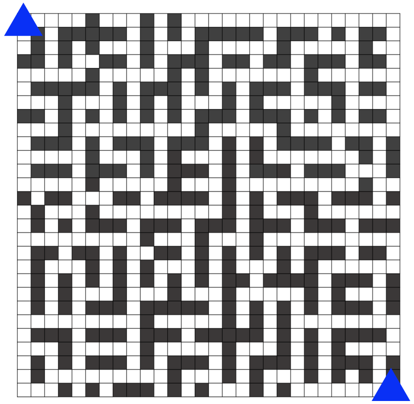
- Many graph problems have variants that are also studied in a Euclidean space.
  - Traveling Salesman Problem.
  - Minimum Spanning Tree.
  - …
- In many cases, the Euclidean variants are easier to solve than the graph variants.



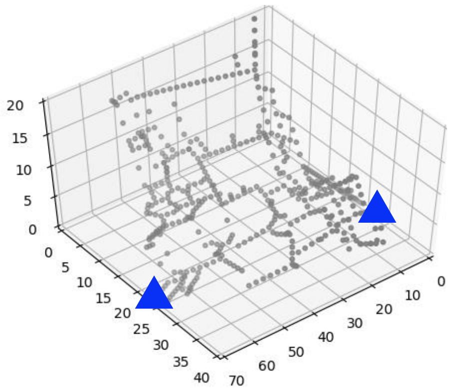


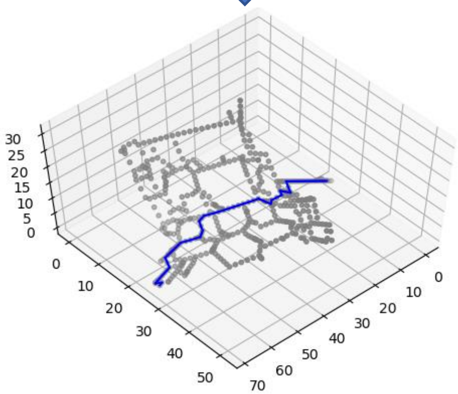
Our idea

Graph problem

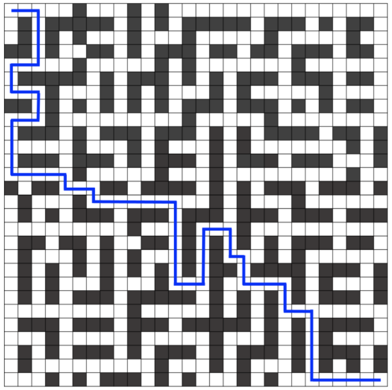


Euclidean problem





Euclidean solution

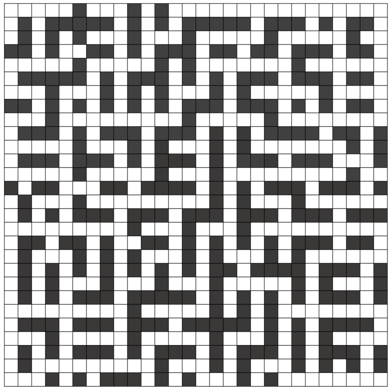


Graph solution

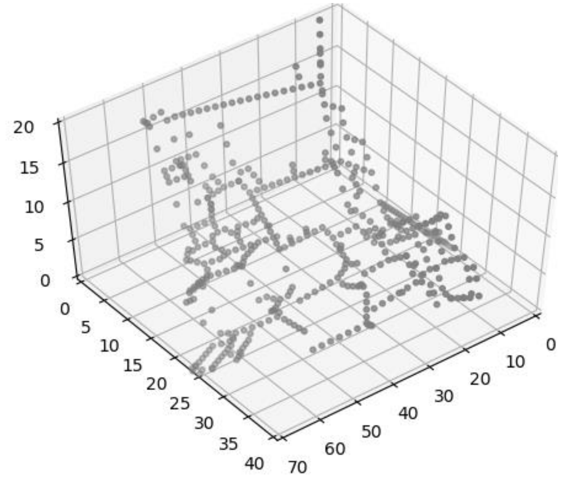
Our idea

- FastMap [Faloutsos et al., 1995; Cohen et al., 2018]
  - Every vertex $v \in G$ is mapped to a point $p \in R^K$.
  - $\text{shortest\_path\_distance}(v_i, v_j) \approx \text{Euclidean\_distance}(p_i, p_j)$.
  - Complexity of the embedding: $O(|E| + |V| \log|V|)$.

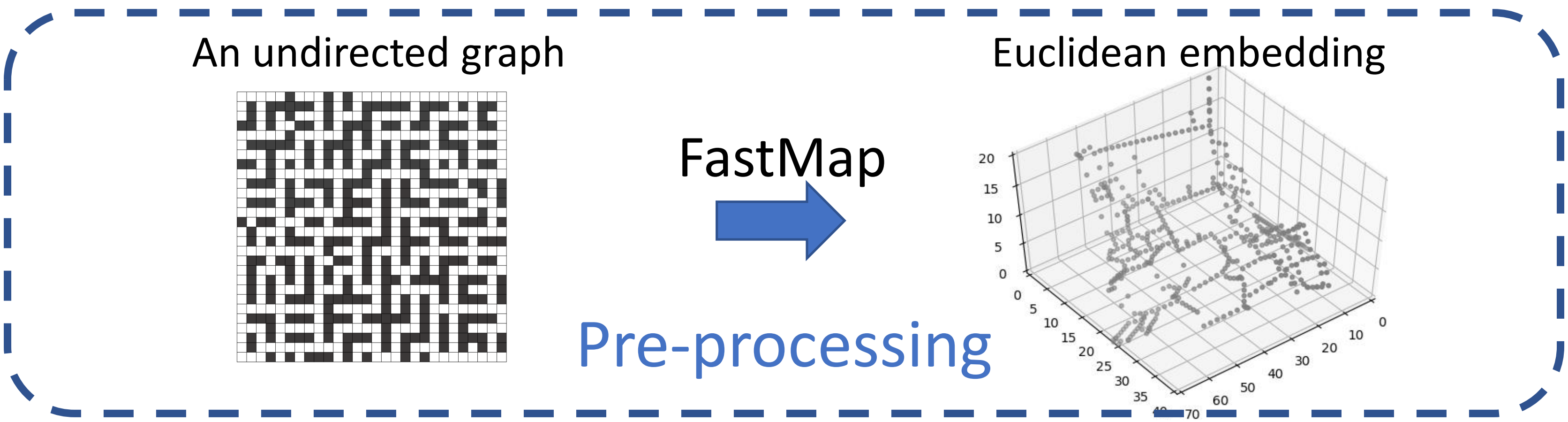
An undirected graph

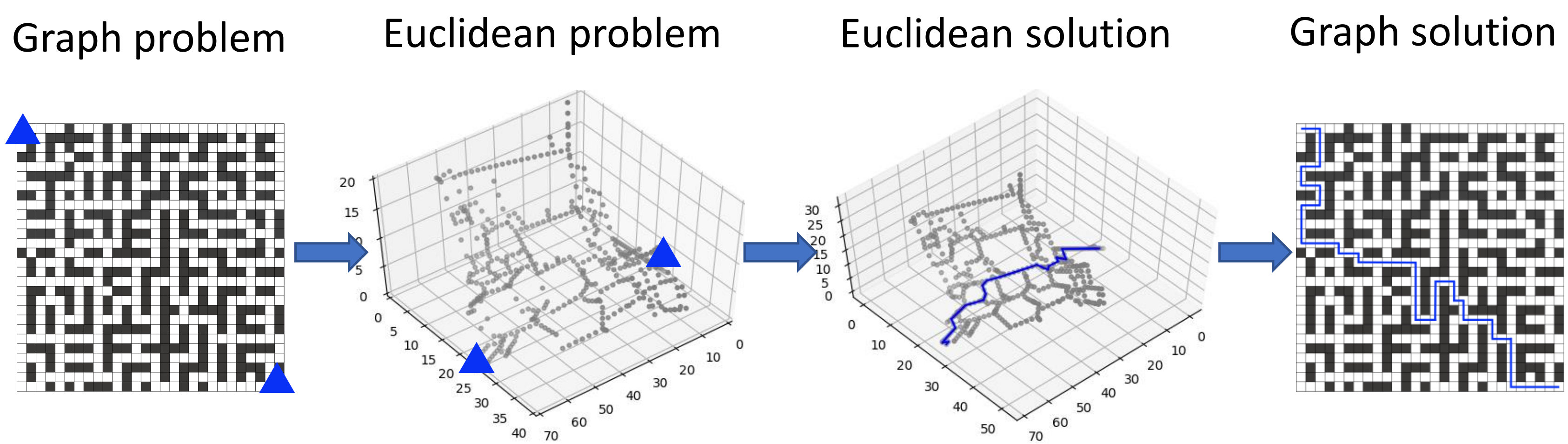


Euclidean embedding



Our idea


An undirected graph
FastMap
Euclidean embedding
Pre-processing


Graph problem
Euclidean problem
Euclidean solution
Graph solution

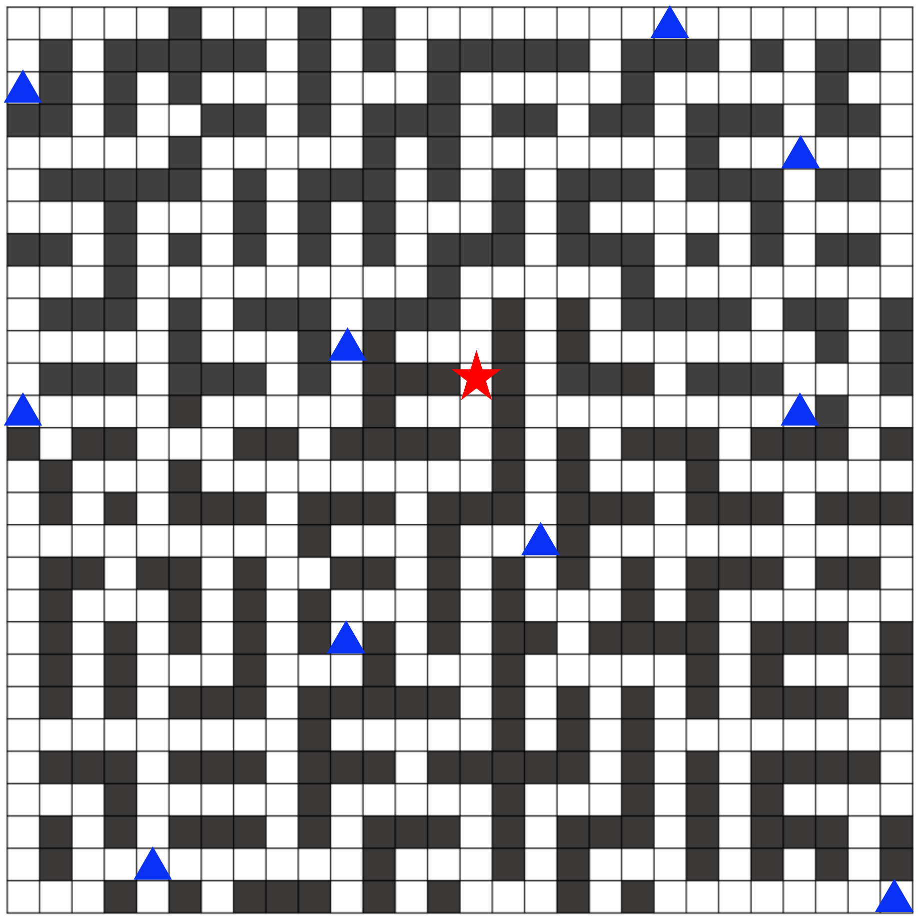
Applications

1. Multi-Agent Meeting Problem
2. Path-Finding Problem

Applications

1. **Multi-Agent Meeting Problem**
2. Path-Finding Problem

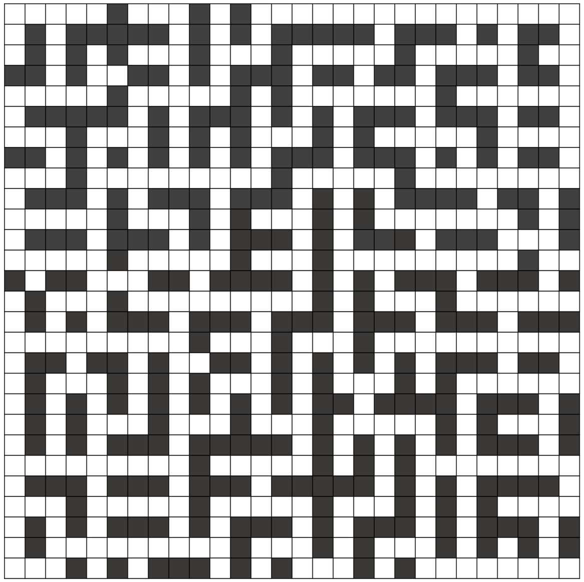
Multi-Agent Meeting Problem

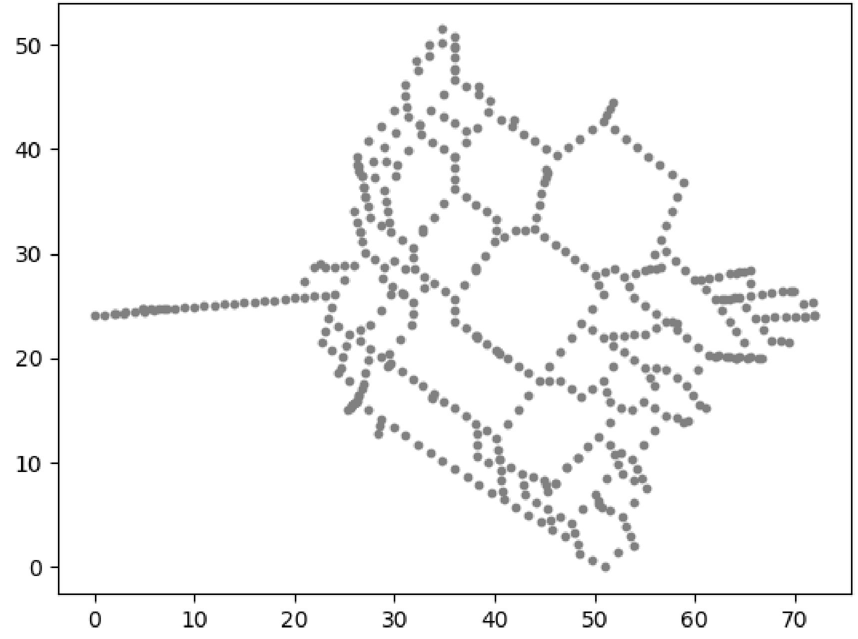


$$\min_{v \in V} \sum_{i=1}^{k} \text{distance}(s_i, v)$$

▲ Start locations ★ Meeting location

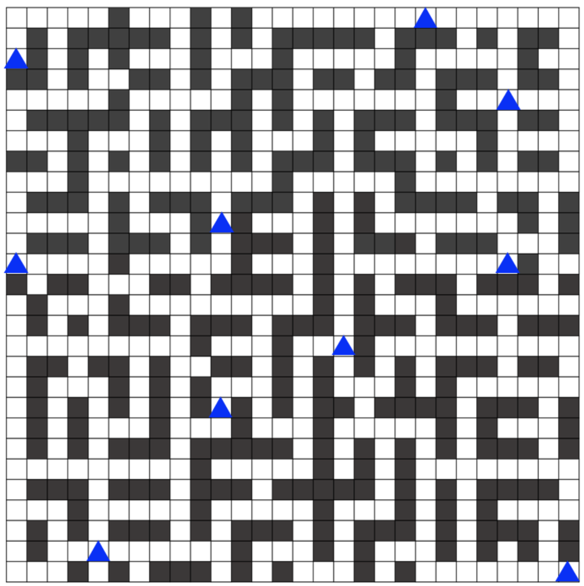
Multi-Agent Meeting Problem

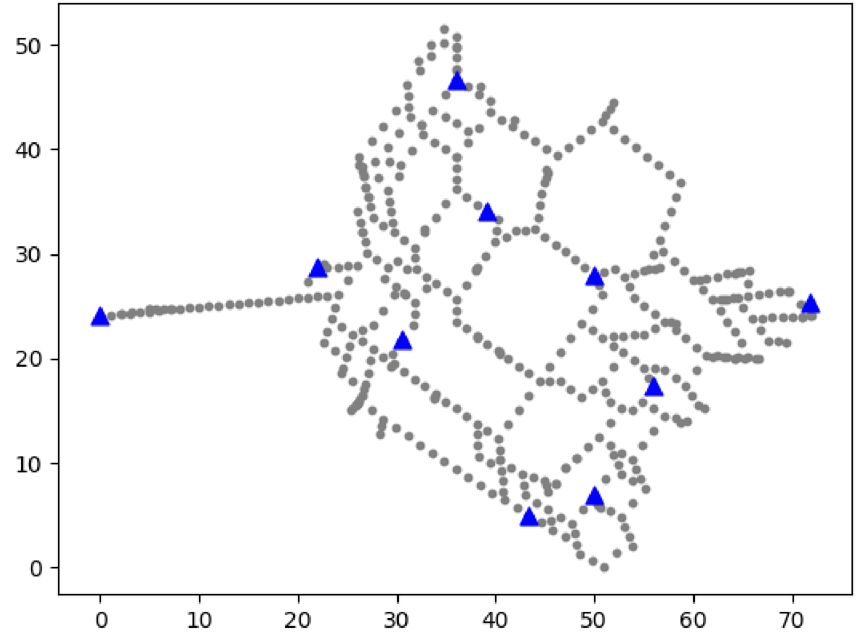




Multi-Agent Meeting Problem

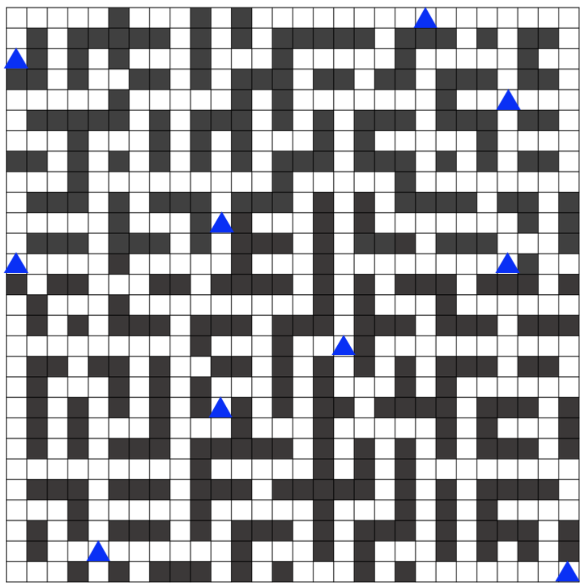
Fermat-Weber problem (1-median problem)

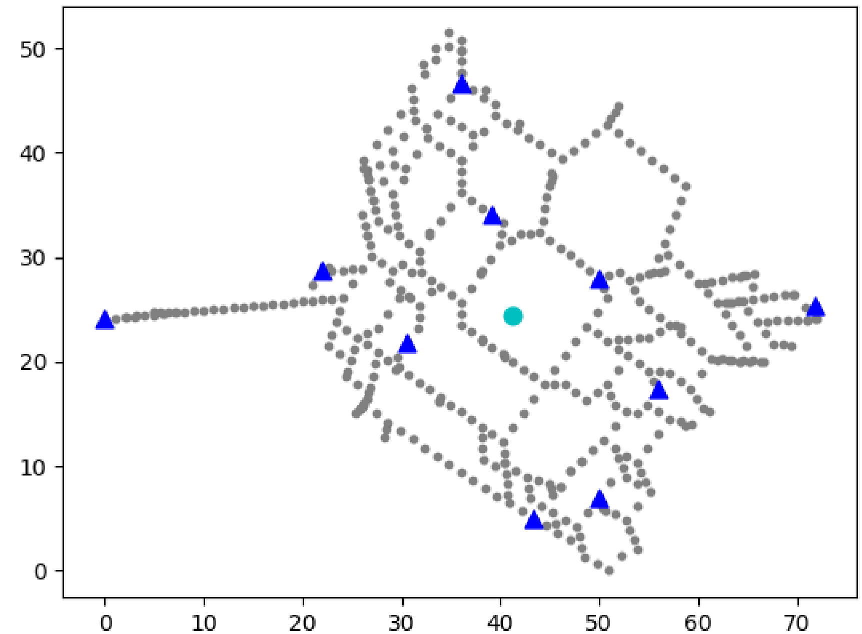




▲ Start locations

Multi-Agent Meeting Problem

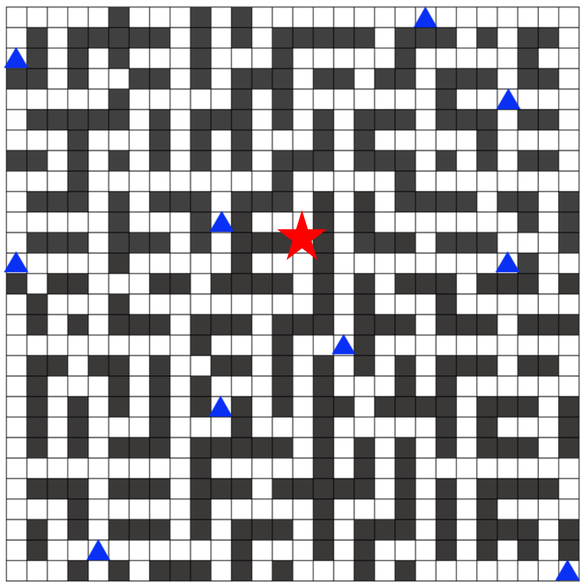


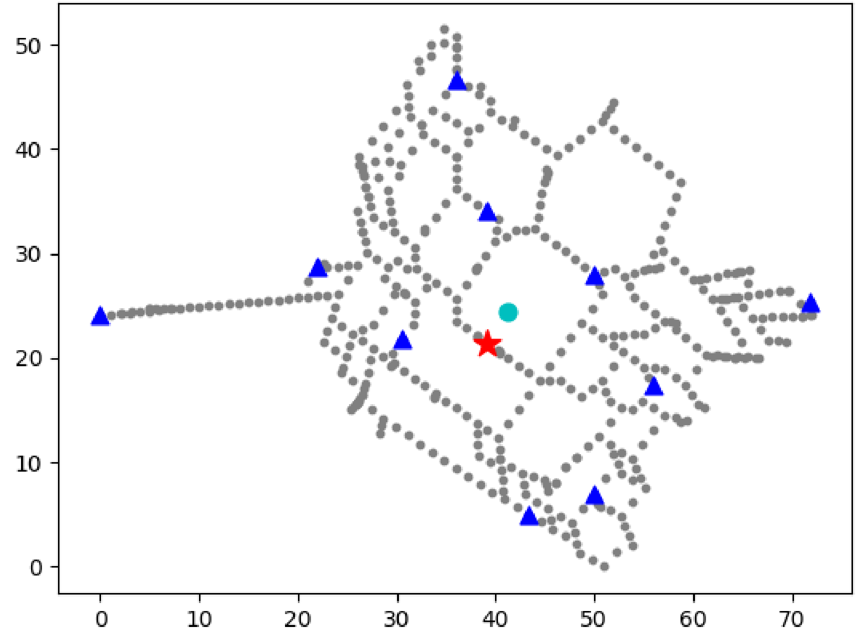


▲ Start locations

● Median point in the Euclidean space

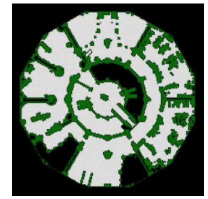
Multi-Agent Meeting Problem

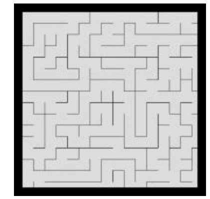


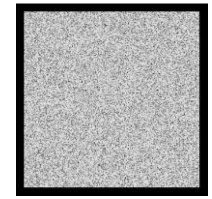


▲ Start locations ★ Meeting location ● Median point in the Euclidean space

Experiments





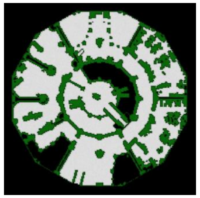
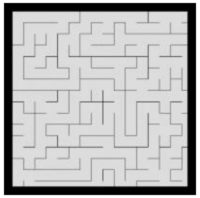
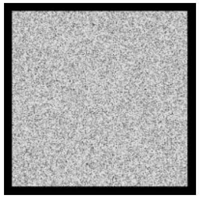


10 start locations

Graph	Suboptimality (%)	Runtime (ms)	Dijkstra runtime (ms)
Game grids	0.22	7	8
	3.00	22	22
Maze grids	6.54	148	177
	6.76	268	362
Random grids	5.96	117	181
	20.53	275	409
General graphs	33.81	1	2
	34.80	4	10

All grids are from [Sturtevant 2012]. General graphs are from [Beasley 1990].

Experiments

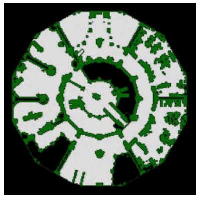
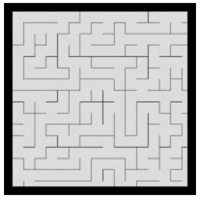
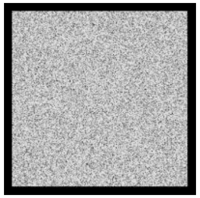




100 start locations

Graph	Suboptimality (%)	Runtime (ms)	Dijkstra runtime (ms)
Game grids	0.22	7	58
	1.32	22	187
Maze grids	4.83	149	1,730
	3.95	268	3,535
Random grids	2.99	118	1,744
	17.81	275	4,051
General graphs	12.53	2	16
	16.79	4	90

All grids are from [Sturtevant 2012]. General graphs are from [Beasley 1990].

Experiments

1000 start locations

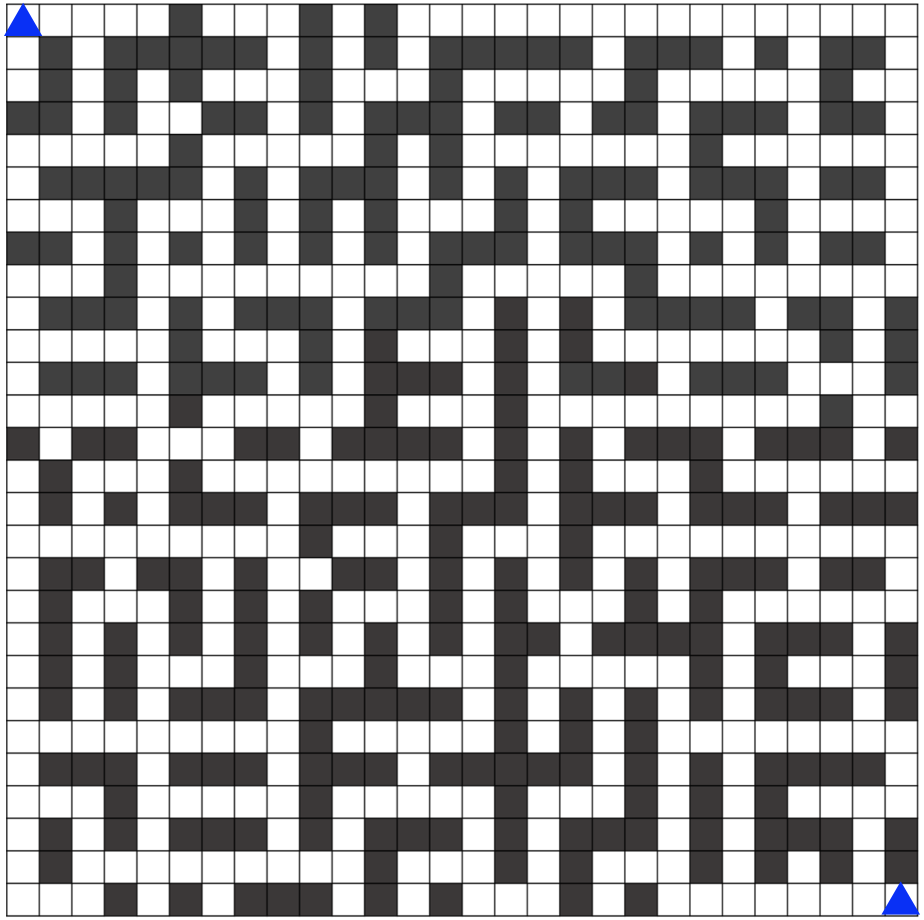
Graph	Suboptimality (%)	Runtime (ms)	Dijkstra runtime (ms)
Game grids	0.07	11	510
	0.98	27	1,841
Maze grids	2.64	155	17,221
	1.17	274	35,450
Random grids	2.69	124	17,633
	17.40	280	40,582
General graphs	9.12	4	144
	15.57	7	810

All grids are from [Sturtevant 2012]. General graphs are from [Beasley 1990].

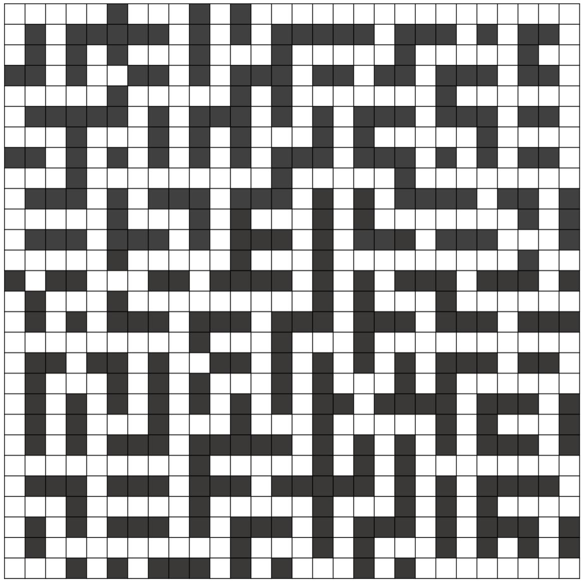
Applications

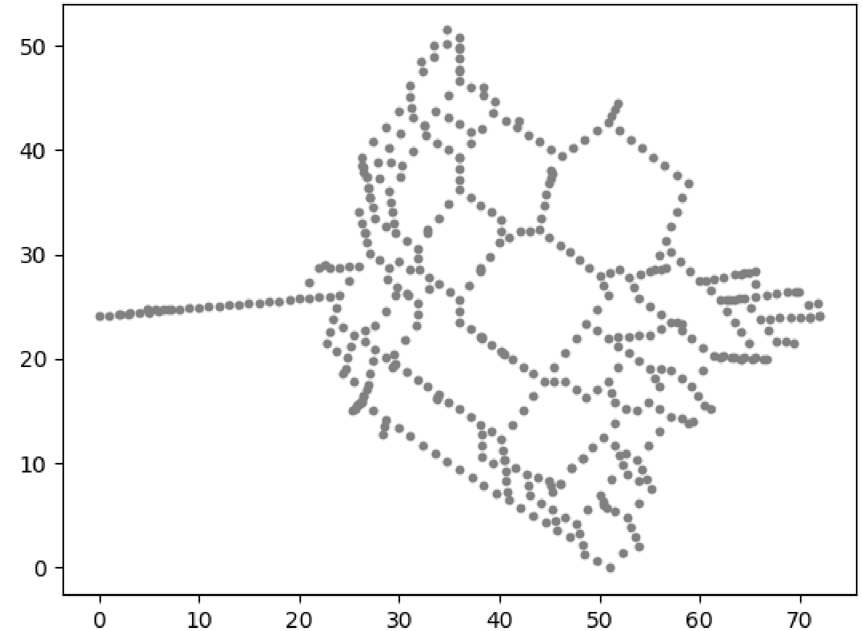
1. Multi-Agent Meeting Problem
2. **Path-Finding Problem**

Path-Finding Problem

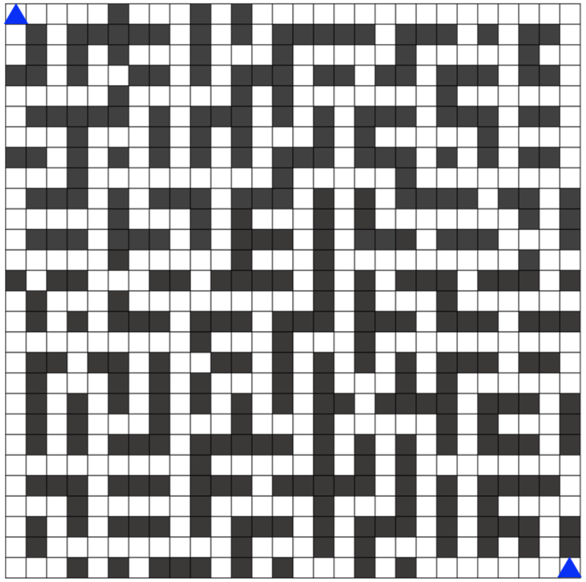


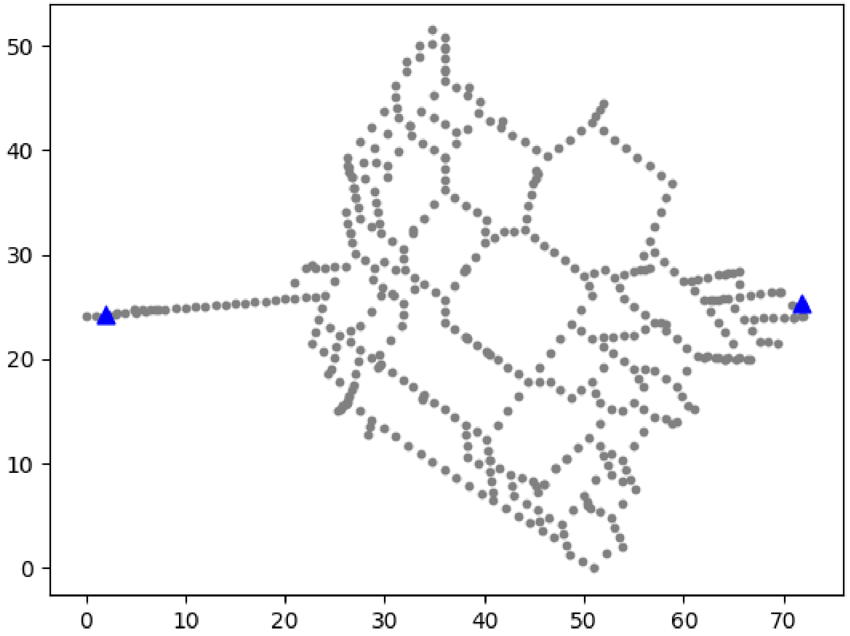
Path-Finding Problem





Path-Finding Problem

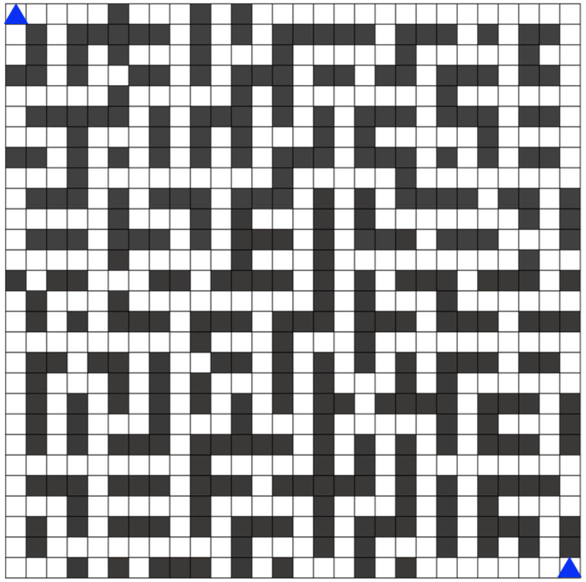


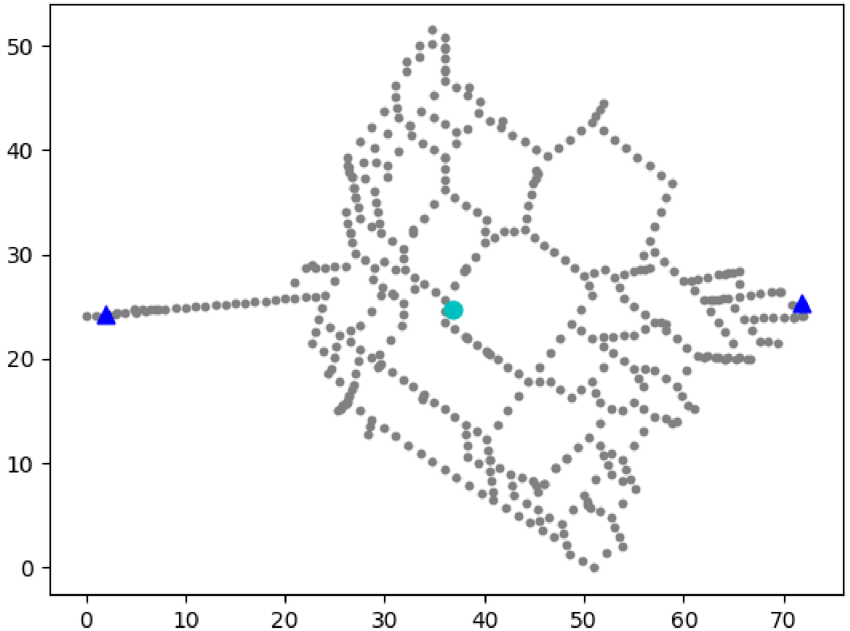


▲ Start and goal locations

Path-Finding Problem

Recursion depth r = 1



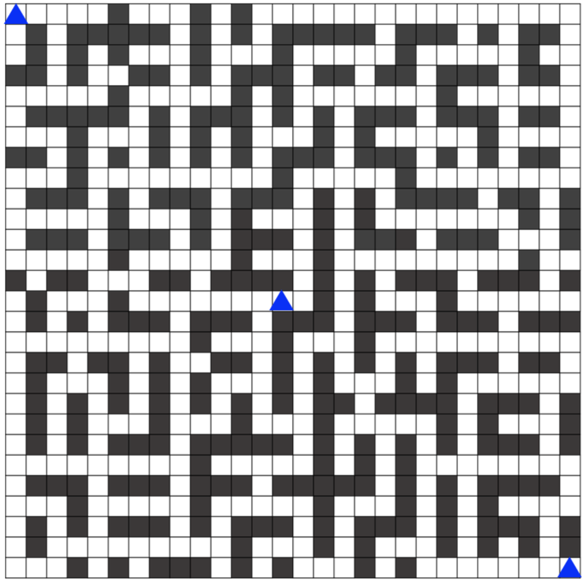


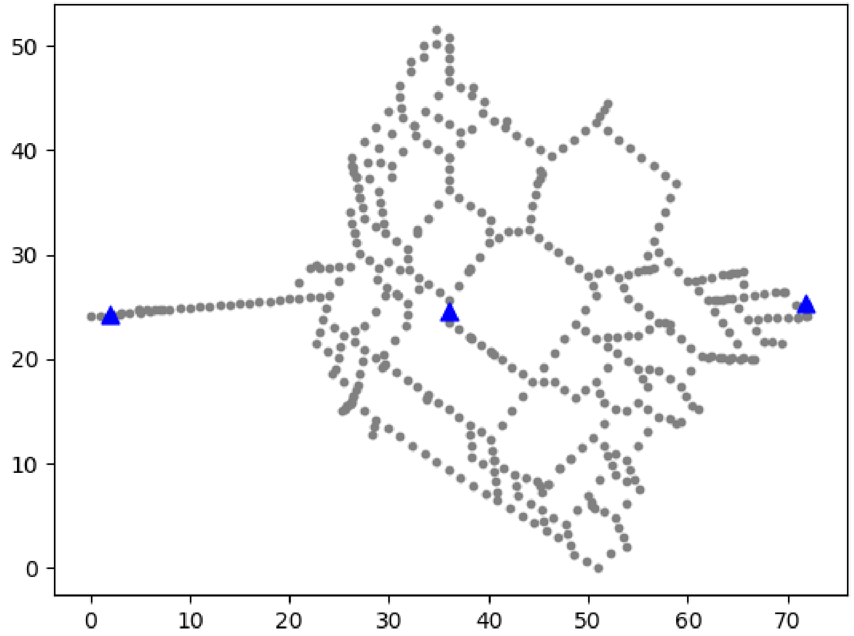
▲ Start and goal locations

● Middle point in the Euclidean space

Path-Finding Problem

Recursion depth r = 1

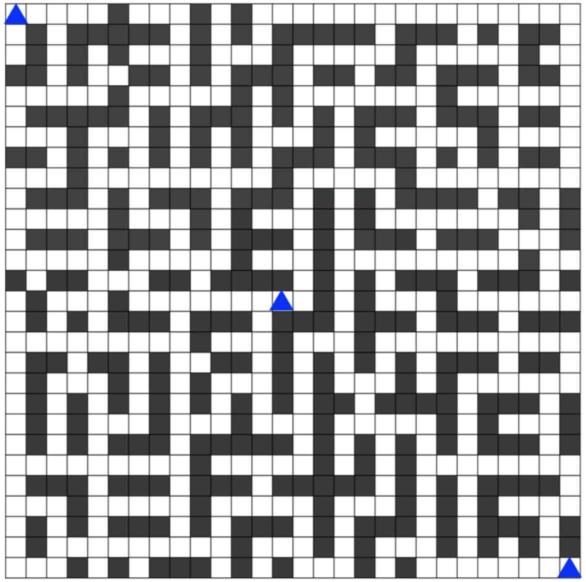


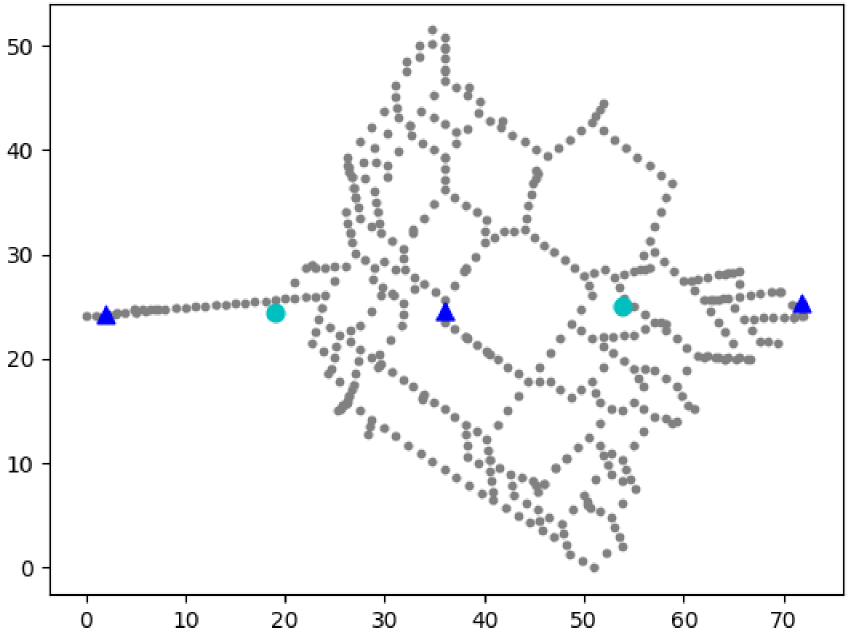


▲ Start and goal locations

Path-Finding Problem

Recursion depth r = 2



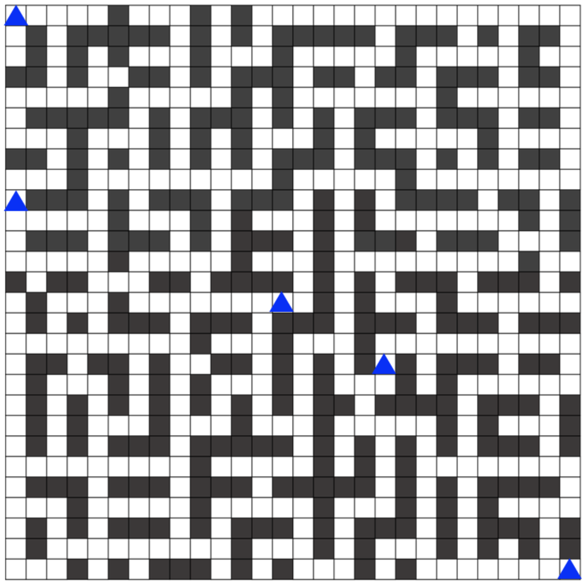


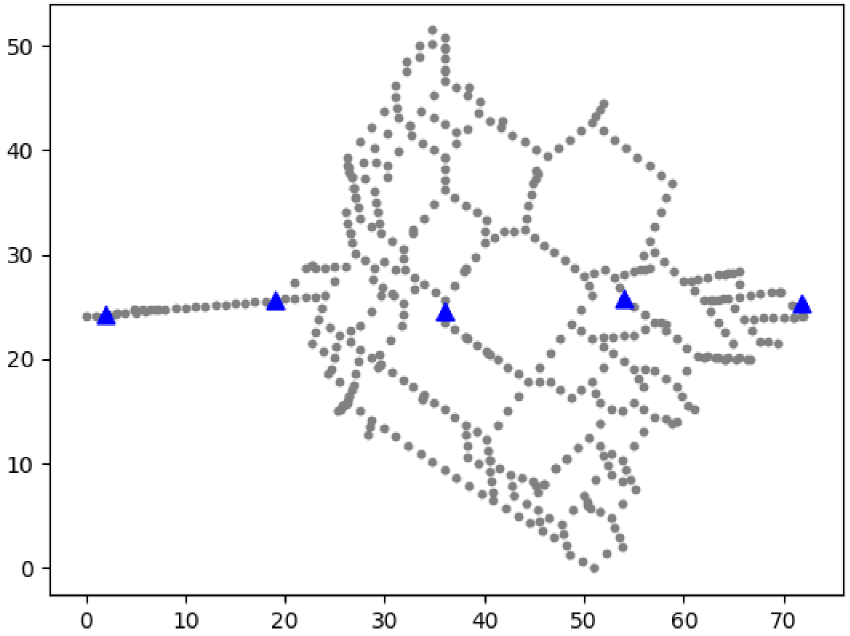
▲ Start and goal locations

● Middle point in the Euclidean space

Path-Finding Problem

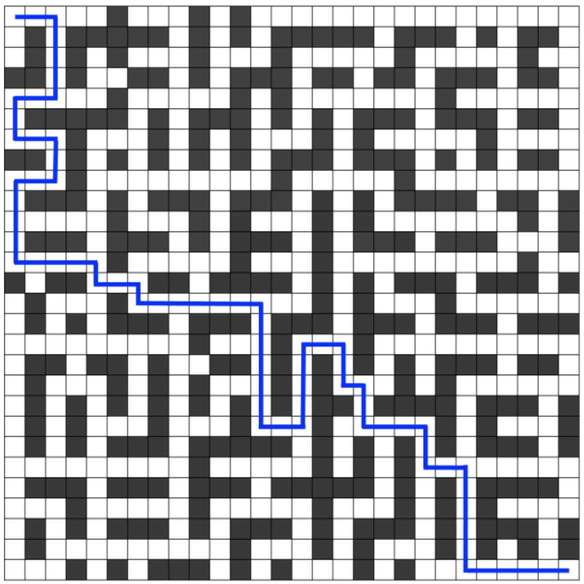
Recursion depth r = 2

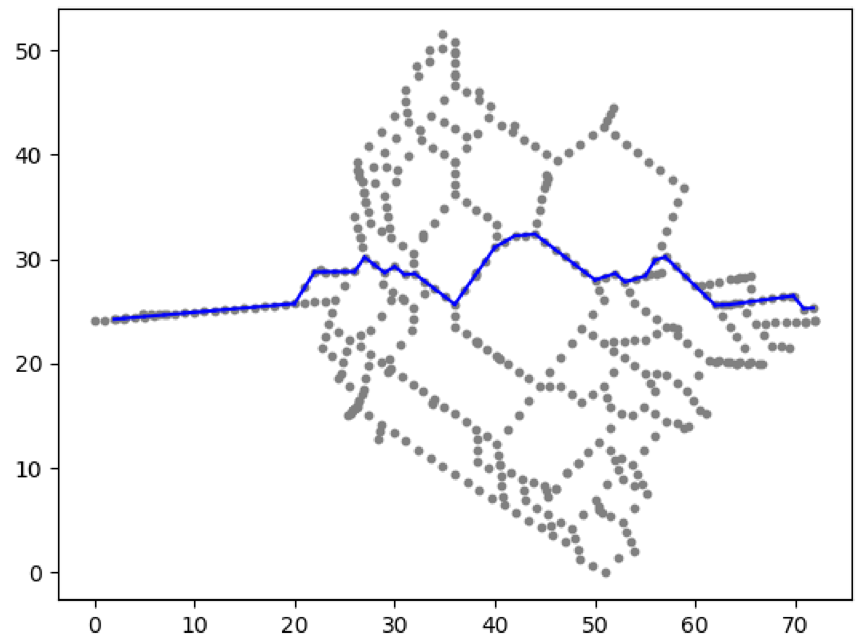




▲ Start and goal locations

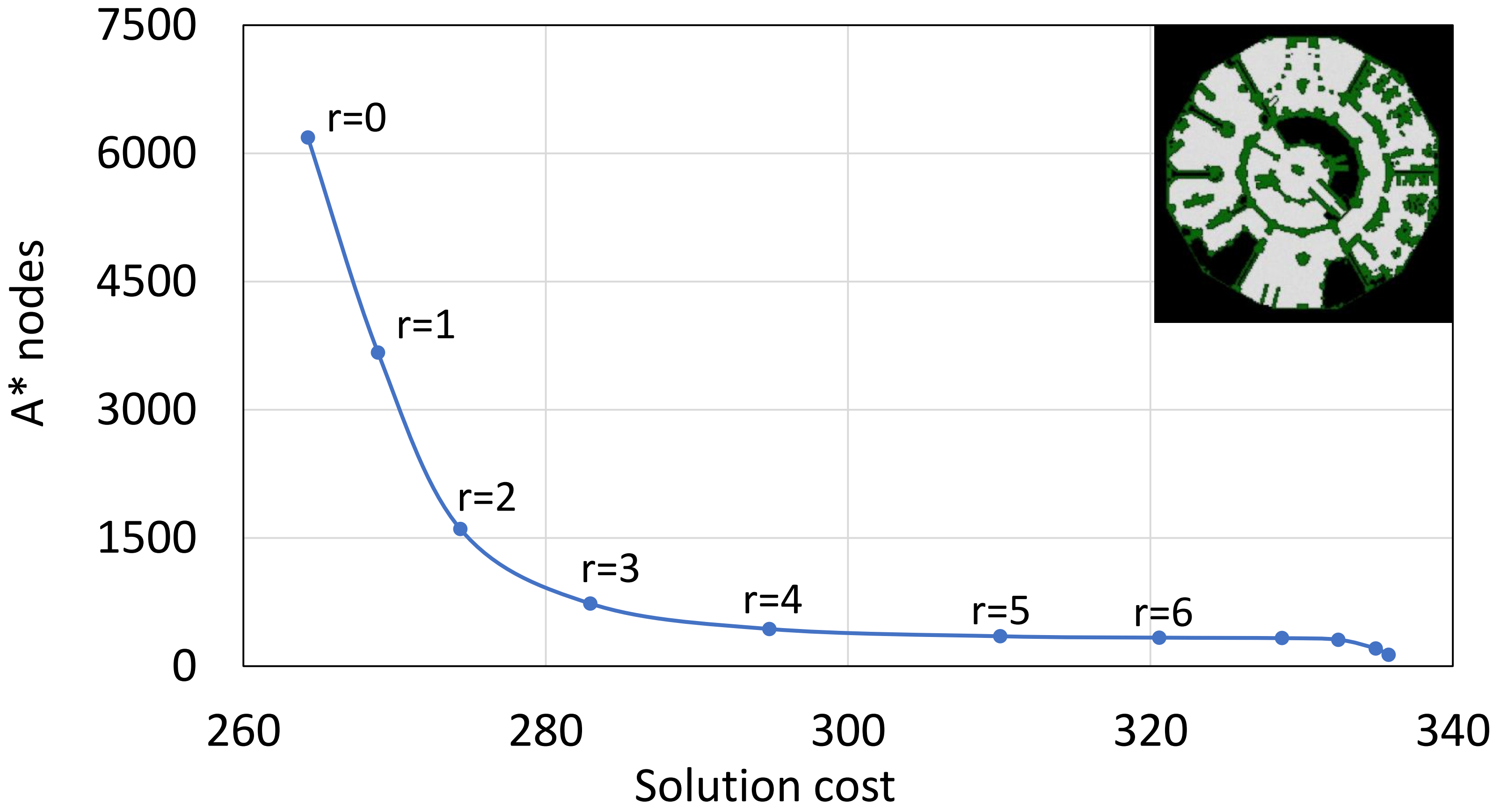
Path-Finding Problem





Experiments

- The runtime is worse than A* search.

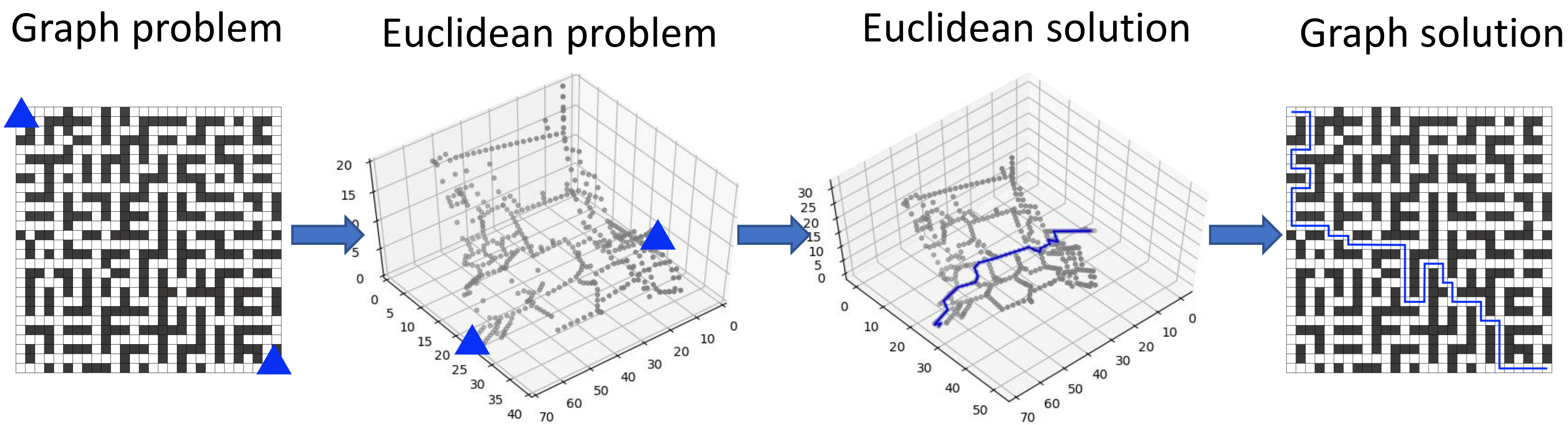


r is the recursion depth.

r = 0 reduces to A* search.

Summary

- Many graph problems have Euclidean variants that are easier to solve.
- Our framework:



- Two applications:
  - The multi-agent meeting problem.
  - The path-finding problem.